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1998 J. Phys.: Condens. Matter 10 9691

(http://iopscience.iop.org/0953-8984/10/43/012)

J. Phys.: Condens. Matter **10** (1998) 9691–9699. Printed in the UK PII: S0953-8984(98)93778-X

Magnetic-field-induced anomalous phase transitions in p-SiGe/Si heterostructures

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Received 29 April 1998, in final form 29 June 1998

Abstract. We present the low-temperature magneto-transport measurements on a two-dimensional hole gas in the p-type modulation doped SiGe alloy confined by an Si barrier. We observe a magnetic-field-induced transition which occurs at high magnetic fields from an integer quantum Hall phase to a Hall insulator phase and finally reenters to a $\nu = 1$ integer quantum Hall phase. This Hall insulator is centred at $\nu = 1.5$ which is unanticipated by the global phase diagram. Scaling analysis and universal phenomena are found to be different at the two transition points. These results are attributed to the unusual energy level scheme in p-SiGe.

Detailed studies in high magnetic field quantum transport in two-dimensional electron system (2DES) has been an active topic shortly after the discovery of the integer quantum Hall effect (IQHE). The physics of electronic transport in the presence of impurities is rich and incompletely understood. The specially interesting critical phenomenon related to the transitions between adjacent quantum Hall (QH) plateaus has been of recent interest. Such behaviour was observed first in an experiment by Wei *et al* [1] that manifested these transitions as plateau transitions in which the width of the ρ_{xx} peaks, (ΔB), and the inverse of the maximal slope of the ρ_{xy} steps, $(\mathrm{d}\rho_{xy}/\mathrm{d}B)^{-1}_{\max}$, at the transition increase with the temperature (T) and follow the relationship $\sim T^{\kappa}$, with $\kappa = 0.42 \pm 0.02$, independent of the Landau level (LL) and sample. This universality has been regarded as a telling signature of a quantum phase transition.

The interest has been renewed recently when the magnetic-field-driven transitions which are not between adjacent QH states have become a subject of debate [2–6]. The transitions have some apparently contrasting features to the inter-QH transitions. Experimentally, for example, it was observed in many experimental groups that in the GaAs-based systems, there is a direct phase transition from an insulator phase (IP) to a $\nu = 2$ integer quantum Hall state (IQHS) with the $\nu = 1$ IQHS missing even at fields as high as 10 T [4]. The transitions for the IP to $\nu = 3$ IQHS at low fields and the $\nu = 1$ IQHS to the IP in the high field regions were subsequently found in a 2DHG in the Ge layer [6]. In general, they are characterized by the transitions from QH states to the (IP) state that terminates the QH series and in the IP, $\rho_{xx} \to \infty$, without a ρ_{xx} peak which appears at the inter-QH transitions. Furthermore, the experimental ρ_{xx} values at the transition point between QH and IP are in support of the existence of the theoretically expected universal critical value. In many cases reported, these transitions show the consistent universal scaling behaviour in the vicinity of the transition point with the characteristic critical exponent κ as mentioned above, for the inter-QH transitions.

Recently, a theoretical global phase diagram (GPD) has been proposed by Kivelson, Lee and Zhang (KLZ) attempting to map out all quantum phase transitions in the quantum Hall regime [7]. This model, neglecting the spin of the particles, predicts the existence of a 'Hall insulator' (HI) between integer and fractional quantum Hall states as well as between primary fractional QH states. An important selection rule which assumed that two delocalized states do not merge together also predicts possible continuous phase transition in the IQHS. A magnetic-field-induced transition from one IQHS to another or to the IP must change in σ_{xy} by a step of e^2/h across each boundary assuming the 'floating up' of the delocalized states above the Fermi level is one at a time. According to the framework of KLZ's principle, a continuous transition from the IQHS to the IP is only possible in the lowest LL. One aspect of the observed missing in the spin-split $\nu = 1$ IQHS at high fields and the direct transition from the $\nu = 2$ IQHS to an IP is inconsistent with the GPD proposed by KLZ. Experimentally, there seems to have no evidence of ruling out the existence of any intermediate phases between the QHS and the IP. It is clear that more intensive exploration of a variety of the phase transitions in 2DES is crucial in establishing the topology of the phase diagram.

In this paper we present the study on a magnetic-field-induced phase transition in the Si-based system which is in some extent different from that found in the GaAs-based system. Due to its low mobility, most investigations in Si were performed for the electron system rather for the hole system. However, recent advances in the state of the art for preparing SiGe layers on (100) Si substrates by MBE or UHV-CVD exploit the possibility for the analogous study for the hole system [8, 9]. In the previous work, it was found in the low temperature transport measurements of high quality p-type Si/SiGe heterostructures that in addition to the anomalous spin-unresolved IQHE, two IPs at $\nu = 1.5$ and $\nu < 0.5$ can be observed in such systems [10, 11]. Similar results have been reported by Dorozhkin *et al* [12] and Coleridge *et al* [13] in which samples prepared by different systems or by different designs are used. Here, we will focus on the phase transition properties of the IP centred around $\nu = 1.5$.

The p-type samples used in the experiment were grown in a multi-wafer UHV/CVD system [9]. In these structures, holes are confined in two dimensions in a SiGe layer (∼40 nm thick) sandwiched between the Si substrate and a top Si cap layer, which is modulation doped with boron from the substrate side of the potential well (10 nm spacer). The SiGe layer is under compressive biaxial stress. The Ge fraction of the alloy is about 12 at.%. Hall bar patterns with 5:1 ratio between the conductivity probes and the Hall probes were etched by standard lithographic techniques. The width of the sample is 10 μm. The magneto-transport measurements were performed by using an He^3–He^4 dilution refrigerator or an He^3 cryostat with a rotating probe. With this probe, the angle θ between the normal of the two-dimensional hole gas (2DHG) plane with respect to the magnetic field can be varied continuously from 0 to 90°. Low frequency AC (∼150 Hz) excitation and phase locking detection systems were employed. All data were obtained with driving current as low as 10 nA to avoid the possible hot electron effect.

Figure 1 shows the magnetic field evolutions of diagonal, ρ_{xx}, and Hall, ρ_{xy}, resistivities at $T = 30$, 65, 160, 420 and 650 mK. Integer quantum Hall resistance is clearly observed, characterized by the integer multiple quantum Hall resistance plateaus in ρ_{xy} and the accompanying minima in the ρ_{xx} traces, but only at the odd fillings, ν. This was shown to be due to the fact that the energy of Zeeman splitting is close to that of Landau splitting [10]. There are no harmonic components in the oscillation spectrum. Together with the correct carrier concentration given by the low-field Shubnikov–de Haas (SdH) oscillation, this implies that the mobility is $\sim$6000 cm^2 V^{-1} s^{-1} for a carrier concentration of 3.8×10^{11} cm^{-2}. An effective mass of 0.27 m_0 was deduced from the temperature dependence of the SdH oscillations.

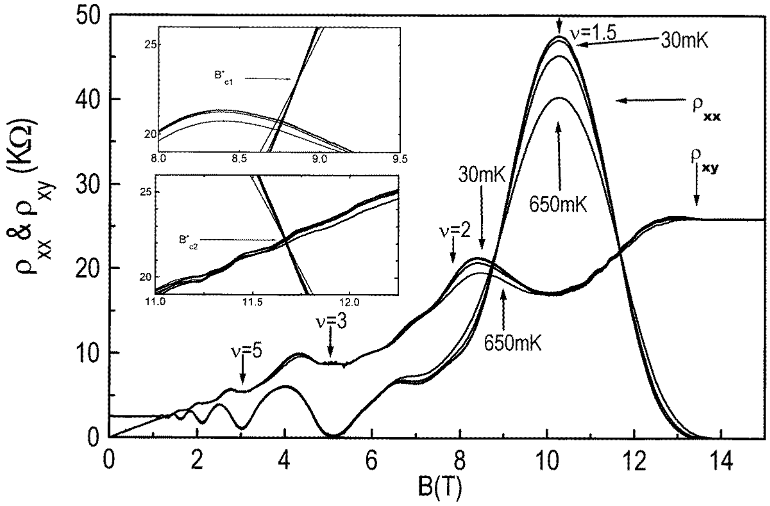

Figure 1. ρ_{xx} and ρ_{xy} versus magnetic field (B) at $T = 30$, 65, 160, 420 and 650 mK. The insets show the magnified view around the transition. In the top-left inset, B^*_{c1} (8.8 T) denotes the critical magnetic field at which the transition from an IQHS to an insulator occurs. In the bottom-left inset, B^*_{c2} (11.7 T) denotes the critical magnetic field at which the transition from an insulator to an IQHS occurs.

One of the main features of these transport measurements is the observed anomalous phase transition at high magnetic fields (>7 T) when the spin-splitting begins to be resolvable. A well developed $\nu = 1$ IQHS below 14 T with vanishing ρ_{xx} and ρ_{xy} quantized to h/e^2 which characterizes clearly the spin-polarized lowest LL state is observed. With decreasing magnetic fields, a distinct magnetic-field-induced IQHS ($\nu = 1$)–IP transition occurs and reenters to the $\nu > 1$ IQHS at lower magnetic fields. Two clear transition magnetic fields, B^*_{c1} and B^*_{c2}, where the ρ_{xx} value is temperature independent, are located at 8.8 and 11.7 T, respectively. The value of longitudinal resistivities corresponding to B^*_{c1} and B^*_{c2} are $\rho^{c1}_{xx} \simeq 0.90h/e^2$ and $\rho^{c2}_{xx} \simeq 0.85h/e^2$, respectively. The temperature independent resistivities with values close to h/e^2 at the transition points indicate a possible universality of this phase transition. For $B^*_{c1} < B < B^*_{c2}$, the 2DHG behaves like an insulator in the sense that ρ_{xx} increases as temperature decreases. An exponential increase in the value of ρ_{xx} and the finite value in ρ_{xy} with decreasing temperature are characteristic of the HI. Notice also that ρ_{xx} tends to saturate for temperature below 100 mK. Thus, the temperature independent points, B^*_{c1} and B^*_{c2}, make up the IQHS–HI–IQHS quantum transition of the system.

Notice first that, at the spin-polarized $\nu = 1$ IQHS to IP transition point, B^*_{c2}, ρ^{c2}_{xx} is very close in value to ρ^{c2}_{xy} as shown in the bottom-left inset of figure 1. The same relation was also observed in the GaAs-based systems [2, 4, 5] and in a 2DHG in a Ge layer [6]. Interestingly, we note that, however, ρ^{c1}_{xy} is found to substantially deviate from ρ^{c1}_{xx} at the transition point, B^*_{c1}, where the transition reenters to $\nu > 1$ IQHS from IP as shown in the top-left inset of figure 1. Consider that, in the neighbouring regime below B^*_{c1}, the even filling $\nu = 2$ is barely implied at appropriate magnetic fields. This anomaly has been checked by tilted field experiments suggesting the interplay of the spin splitting and adjacent LLs which resolves only odd filling factor states except that the lowest LL is left with single spin-up states. This is the consequence of the fact that the LL splitting is approximately equal to that of the spin at the same magnetic field of this unique system [10]. The SdH data at different tilting angles, θ, in figure 2, show that the $\nu = 2$ dip in ρ_{xx} is increasingly resolved as the tilted angle is increased. This is the result of the spin splitting being larger than the LLs. As a consequence, a highly anisotropic g-factor with the component normal to the surface $g_\perp \sim 6$–8, was estimated [10]. The g-factor anisotropy of the 2DHG has also been observed in p-SiGe samples by Whall *et al* [14], and by Glaser *et al* [15] who found from optically detected magnetic resonance a g-factor of 4.5. In this situation, the adjacent LLs with opposite spin are closer spaced than the spin splitting itself. With a line broadening comparable to or larger than these closely spaced pairs, the twofold spin and LL degeneracy are no longer resolved. The unresolved spin levels at $\nu = 2$ become more separate in energy as the tilted field increased since it depends on the total strength of the magnetic field while the LL depends on the normal component of the field.

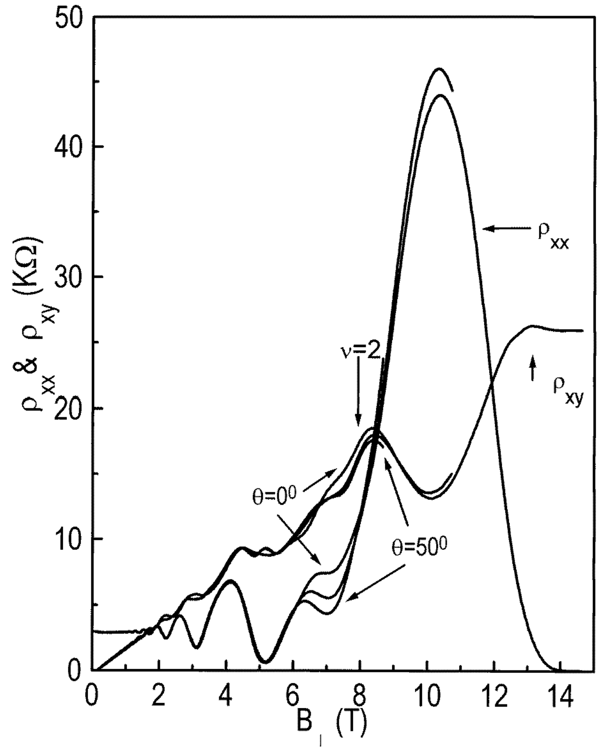

Figure 2. Magnetoresistance as a function of magnetic field normal to the interface with tilting angle as parameter at 300 mK for the sample shown in figure 1. The variation at the $\nu = 2$ dip indicates that the spin degeneracy is resolved as the tilt angle increases.

Next, at the phase boundaries, B^*_{c1} and B^*_{c2}, we found that the conductivity also exhibits similar transition behaviour. Figure 3(a) shows the diagonal, σ_{xx}, and Hall, σ_{xy}, conductivities obtained according to the matrix transformation relations:

$$\sigma_{xx} = \rho_{xx}/(\rho^2_{xx} + \rho^2_{xy}) \qquad \sigma_{xy} = \rho_{xy}/(\rho^2_{xx} + \rho^2_{xy}). \tag{1}$$

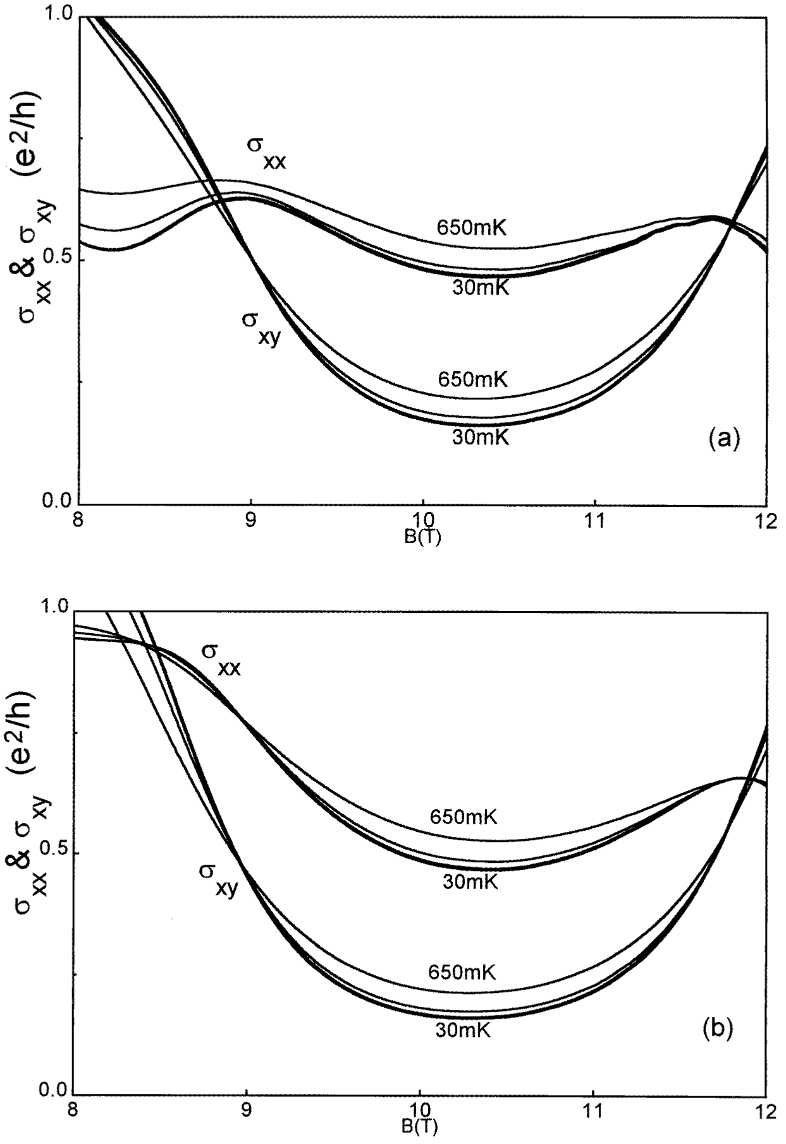

Figure 3. (a) Matrix transformed σ_{xx} and σ_{xy} using measured ρ_{xx} and ρ_{xy} data shown in figure 1. (b) σ_{xx} and σ_{xy} calculated from measured ρ_{xx} data and the classical value of ρ_{xy} ($=B/ne$) by matrix inversion.

At these two transition points, the corresponding σ_{xy}^{c1} and σ_{xy}^{c2} are clearly identified to be temperature independent with a value around $e^2/2h$ suggesting a universality of the phase transition. While the diagonal resistivity at the phase boundaries has a well defined temperature independent value of about h/e^2, the matrix transformed diagonal conductivity, σ_{xx}, does not show a clear transition in an usual sense, i.e., a temperature independent critical point separates the two conducting phase regions. In fact, at the 'transition', or phase boundary, the conductivity merely shows a minimum of activation energy. This discrepancy can be removed if the diagonal conductivity is obtained by converting the resistivity tensor with the classical value of Hall resistivity ($\rho_{xy} = B/ne$) which is shown in the inset of figure 3(b). Apparently, near the IP-IQHE phase boundaries, the matrix converted temperature dependent behaviour of σ_{xx} is caused by the associated rather large anomalous temperature dependence of the Hall resistivity. We suggest that this finding may also be the explanation of the similar difficulty observed in GaAs-based system [4].

Several peculiar behaviours can be deduced from our observed quantum phase transitions of the 2DHS in SiGe quantum well. First, let us consider the anomalous temperature evolution of the Hall resistivity at the fields slightly higher than the value where it should be $\nu = 2$ (at $B = 7.8$ T). The value of ρ_{xy} shows a very large discrepancy to $h/2e^2$. The fact that ρ_{xy} reaches 0.83 h/e^2 as temperature decreased to 30 mK unambiguously indicates that there is only one spin-resolved LL extended state contributing to the conduction, and the other spin-resolved LL extended state is missing. However, the corresponding ρ_{xx} is still finite. On the bases of these observations, one may infer that the missing LL extended state may either become localized due to exceedingly large spin splitting at this point that pushes the lowest LL to the localized energy regime, or float up the upper LL above the Fermi level. Experimentally, the prediction that the extended state will float up with decreasing magnetic field at constant or increasing disorder has been confirmed [16]. If we adapted the idea of the floating up of extended state in the limit of $B \rightarrow 0$, the succeeding observation of an insulating phase centred at $\nu = 1.5$ at high magnetic fields can thus be understood, because the missing extended state can be partly restored when the magnetic field increases. However, our experimental condition is far more complicated than the known theoretical considerations. A direct comparison between the theory and experiment is not appropriate at the present time. Nevertheless, the unusual energy level scheme resulted from the crossing of LLs with opposite spins in the p-SiGe quantum well is believed to be closely related to the intricate phase transitions.

The IP near $\nu = 1.5$ shows the characteristic definition of a Hall insulator, namely, $\rho_{xx} \rightarrow \infty$ and nearly normal ρ_{xy}. However, the observation of this phase transition clearly violates the KLZ prediction. Recently, a number of observations that reflect the violation of the KLZ global phase diagram have been reported. For a 2DES in GaAs/Al$_x$Ga$_{1-x}$As heterostructures, the transition that starts from the IP to the $\nu = 2$ IQHS with $\sigma_{xy} = 2e^2/h$ (this will be called the $I \rightarrow 2$ transition) and reenters to the IP was observed [2–4]. Applying this transition to the KLZ framework, it can be reconciled by assuming that in a low-mobility 2DES, spin states are energetically degenerate or nearly degenerate. More recently, in a similar system, an $I \rightarrow 1 \rightarrow I$ transition was observed, and if an $I \rightarrow 2$ transition could be explained this way, then the $1 \rightarrow I$ transition would have been observed for better quality samples as the magnetic field is increased [5]. Succeedingly, the $I \rightarrow 3 \rightarrow 2 \rightarrow 1 \rightarrow I$ was observed in a 2DHS in a Ge layer [6]. Based on these results, it is possible that the magnitude of the spin splitting and the degree of disorder are crucial in establishing the topology of phase diagram. Considering the phase transition in the present work, although the reasons for the discrepancy between the observation and the proposed GPD remains unclear, it is worth noting that one of the basic assumptions of the KLZ selection rule is that two delocalized levels do not merge. Therefore, the boundary between the $\nu = 1$ and $\nu = 2$ IQHSs which occurs at the filling $\nu = \frac{3}{2}$, will become unclear for the case of the overlapping of the two extended states with opposite spins. Another question of whether or not there is a phase transition from spin-resolved IQHS to a spin-unresolved IQHS is evoked by the recently observed IP–IQHS transitions. Our work presented here may imply that an insulating region possibly exists between $\nu = 1$, and $\nu = 2$ under some strict conditions.

Since in a two-dimensional system the conductivities in the scaling regime are generally expected to be in the same universality [17], we thus investigate the transition evolutions

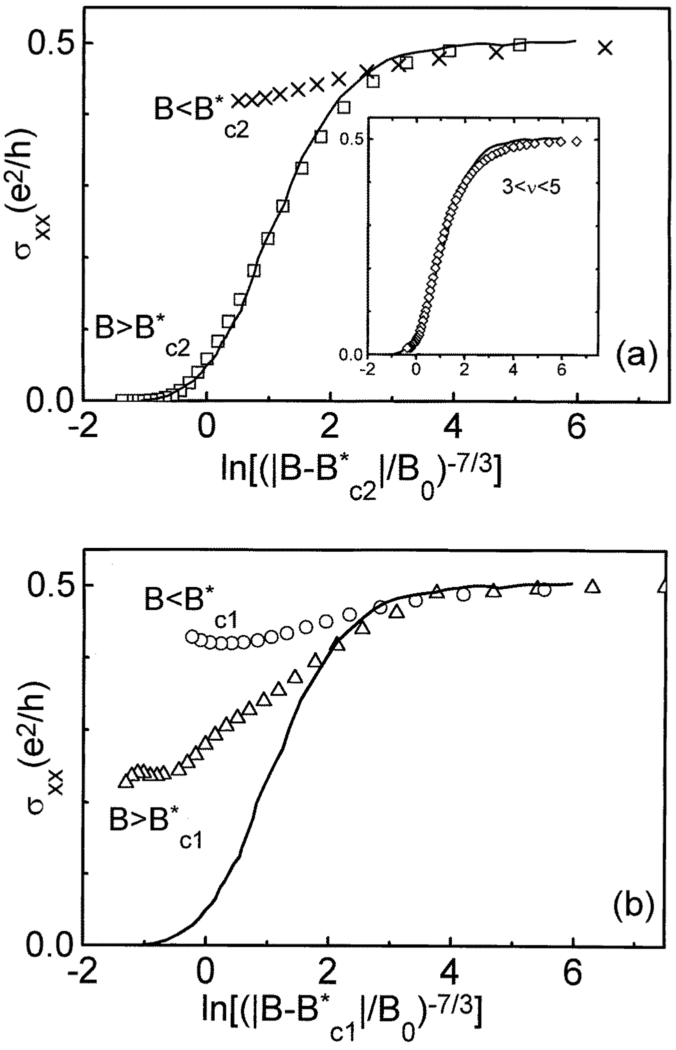

Figure 4. The scaling plot for $\sigma_{xx}(B)$ at $T = 65$ mK. (a) The coinciding curve is for $B > B^*_{c2}$ (□). The (×) curve is for $B < B^*_{c2}$. Inset: the scaling plot for σ_{xx} of the transition between $\nu = 3$ and $\nu = 5$. (b) The (△) curve is for $B > B^*_{c1}$. The (○) curve is for $B < B^*_{c1}$. Solid lines in both (a) and (b) are the theoretical value [14].

of the conductivities according to the recently proposed scaling model [18]. A central point of this model is that both integer and fractional plateau transitions in the QHE are due to quantum percolation. The σ_{xx} were extracted from experimental data by subtracting the edge current contribution from the parent Hall liquid. Such an analysis was done by McEuen *et al* [19] for the transition between the second and the third Hall plateau. The model based on quantum percolation leads to the single parameter scaling relation: $\sigma_{xx} = \ln(\mid B - B_c \mid /B_0)^{-\frac{7}{3}}$, where B_0 is the only adjustable parameter. For our results, the scaling of σ_{xx} is very good for the transition between $\nu = 3$ and $\nu = 5$ plateaus. This is shown in the inset of figure 4(a) where our experimental σ_{xx} has been scaled for its height to be $e^2/2h$ and then fitted to the numerical result via the single parameter B_0. For the transitions near B^*_{c1} and B^*_{c2}, only the transition to the spin-resolved $\nu = 1$ IHQS above B^*_{c2} can be well fitted, as shown in figure 4(a). On the other hand, as shown in figure 4(a) and figure 4(b), the scaling of σ_{xx} for $B < B^*_{c2}$, $B > B^*_{c1}$ and $B < B^*_{c1}$ were all poor. The fact that these poorly fitted regions fall in the anomalous regimes discussed above may again be related to the unusual energy level scheme presented in the p-SiGe quantum well.

Finally, we consider an interesting relation between the value of ρ^c_{xy} and the possible universal value of ρ^c_{xx} at the transition points. The accompanying ρ^{c2}_{xy} is very close in the value of ρ^{c2}_{xx} at B^*_{c2} for $T \leqslant 420$ mK while ρ^{c1}_{xy} is found to be substantially at least 10% less than ρ^{c1}_{xx} at B^*_{c1} even at the lowest temperature (~30 mK). Notice that B^*_{c1} characterizes the IQHS to IP and B^*_{c2} indicates the IP to IQHS transitions both existing at high magnetic fields (>7 T). For the transitions reported previously, by tuning the carrier concentration and hence the magnitude of disorder, it was found that ρ^c_{xx} is approximately equal to ρ^c_{xy} and both appear to saturate at the value of quantum resistance h/e^2. It should be pointed out that while the samples used in these studies are diverse in the extent of disorder, the width of spacer, doping positions or even the types of carrier, the results show unambiguously a universal phenomena for the IP–QHS transitions. Thus, on general grounds, one indeed expects universal phenomena existing in different classes of systems. Our observation of two clear critical transition regimes with diverse relations for the ρ^c_{xx} and ρ^c_{xy} at high magnetic fields warrants a supplement for further theoretical investigations.

In summary, we observed that at high magnetic fields a transition from an IQHE phase to an insulating phase occurs and reenters to the $\nu = 1$ IQHE state in a 2DHG in p-SiGe. Scaling behaviour due to quantum percolation is only observed as the transition evolves to the spin-resolved $\nu = 1$ IQHE state at higher magnetic fields. The discrepancy in both the scaling behaviour and the universal phenomena at the two transition points is attributed to the unusual energy level scheme in p-SiGe. Our data suggest that a new magnetic-field-induced metal–insulator transition exists between the higher spin-mixed LL and the lowest spin-resolved LL. These results are not consistent with the existing theoretical predictions and the proposed GPD. A systematic consideration of the spin of the system in any applicable theory is clearly needed for the magnetically induced phase transition reported in this paper.

Acknowledgment

This work is partly supported by the National Science Council of the Republic of China.

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